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Random External Fields

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The various theoretical considerations for the effects of quenched random fields (RF) on second-order transitions as well as the experimental situation are briefly reviewed. Some of the physical realizations of the RF models are discussed, with an emphasis on solid-state first-order transitions in impure systems. The physical arguments for the RF effects in the bulk as well as on phase interfaces are discussed. In the latter case it is suggested that scattering experiments can probe the details of the interface fluctuations. The role of long relaxation times and metastability in Ising RF systems is emphasized.

KEY WORDS: Random fields; dimensional reduction; disordered systems; metastability.

1. INTRODUCTION AND REVIEW

Quenched random fields (RF),[1,2] that couple linearly to the order parameter, have a marked effect on regular second-order phase transitions. As shown by Imry and Ma[3] (henceforth abbreviated as IM), the long-range order is destroyed below a lower critical dimension (LCD), d_l. d_l is four[3] for continuous systems and two[3] or three[4-8] in Ising-like ones. For $d > d_l$, the critical behavior is strongly modified from the pure case. An interesting property is the dimensional reduction.[9-14] The d-dimensional RF model is argued to behave like the $d - \delta$ dimensional pure model. δ appears to be equal to 2 for continuous symmetry systems and for RF Ising models (RFIM) at $d = 6$. The behavior of the RFIM is not yet clear: Either

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$\delta = 2$ always, or $\delta = 2$ for $4 \leqslant d \leqslant 6$ and changes continuously to $\delta = 1$ at $d = 3$, or $\delta = 2 - \eta_{d-\delta}$, where η_d is the usual critical exponent at dimension d.[2,13] Recently $\delta = 2 - \eta_{d-\delta}$ was argued to apply also for continuous symmetry systems.[13] Clearly, whatever are the detailed properties of the RFIM, they are qualitatively more different from the pure case than most other perturbations. The dimensional reduction is of interest in other branches of physics as well.[6,7,12,14] We shall also emphasize in this paper the interesting dynamical behavior of the RFIM which includes long relaxation times for long-wavelength motion, metastability, and irreversible behavior.[15-25] These properties resemble very much those of spin glasses, and, in fact, the RFIM may show similar types of spin ordering.[3,26,27]

While the first realization of an RF model in the literature is, as far as we know, the work by Larkin[1] on the pinning of the vortex lattice in type-II superconductors, we can now consider many other examples of RF systems. These include magnetic,[3,26] ferroelectric and displacive,[28-31] lattice[32] (including C-I transitions), electronic (CDW[33,34] states) systems, and, notably, solid state first-order transitions,[35] which deserve, in our opinion, much more study.

The effect of the RF is manifest not only in bulk properties but also in the properties of the interface[36,37] between, say, + and − domains in the RFIM. It turns out[4] that the RF induces important interface fluctuations[4,32,38-40] and additional roughening.[41] This can be examined by scattering equilibriums which we shall also briefly discuss here. We shall also emphasize the unusual line shapes in the critical scattering, expected[4,6,38] and found[16,18,20-25] in RF systems.

After the qualitative arguments and $O(\epsilon)$ RG calculation around $d = 6$ (which is the same as $d = 4$ in the pure model) by IM,[3] Grinstein[9] has considered the $O(\epsilon^2)$ terms and found that d is replaced by $d - 2$ in the exponents and the hyperscaling relations. Aharony, Imry, and Ma[10] attempted to prove that the $6 - \epsilon$ expansion for the RFIM is indeed identical to the $4 - \epsilon$ one for the pure model to all orders in ϵ. This complete calculation has been first done correctly by Young.[11] Parisi and Sourlas[12] first introduced the supersymmetric formulation for this problem, which enabled them to present the above proof in a much more elegant way. This has created some interest in the problem[6,7,14] by the high-energy community. Pytte, Imry, and Mukamel[4] attempted to obtain $d_l = 3$ and to make an ϵ expansion for $d = 3 + \epsilon$ for the RF interface model[38] which they introduced, using the replica method. $d_l = 3$ is consistent with a dimensional reduction $\delta = 2$. The qualitative aspects of this result which do have difficulties (see Section 3) were discussed by Binder, Imry, and Pytte.[5] The $d_l = 3$ result and the $3 + \epsilon$ expansion were then obtained more generally and in a more elegant fashion using the supersymmetry method, by Kogon and Wallace.[6] Later, Cardy[7] has derived the result for general d, Cardy and Boyanovsky[42] considered the dynamics, and Niemi[8] has also found the $d \to d-2$ correspondence in the supersymmetric picture. However, Grinstein and Ma[39,40] and Villain[32] have reconsidered the RF interface problem with a physically very plausible RG procedure, and found $d_l = 2$. Their results are also supported by numerical transfer-matrix computations by Fernandez *et al.*[43] Earlier Monte Carlo[44–46] simulations[15] of the same model were inconclusive but have served to point out and demonstrate the importance of dynamics. Long-lived metastable states exist for this problem and make it quite difficult to achieve equilibrium. An interesting idea of constructing an annealed model so that its statistical mechanics will be identical to that of the RFIM was pursued by Schwartz.[13] The ensuing annealed model is quite complicated, having long-range correlations, but Schwartz has claimed that by considering the leading singular behavior of the correlation functions one can show that the RFIM has the same critical behavior as the pure Ising model in $d' = d - 2 + \eta(d')$ dimensions, in agreement with a qualitative argument in Ref. 10—see Section 3. This does not agree with the ϵ expansion, but it may be argued that the perturbative treatments may be hampered by generalized Griffiths-type singularities as function of the RF.[45,47] Recent transfer-matrix studies of finite size $d = 2$ RFIMs by Fernandez and Pytte[48] confirm the exponential dependence of the correlation length on the inverse square of the RF amplitude, typical to an LCD situation. This is another strong indication that $d_l = 2$ in *equilibrium*.

The experimental situation, while giving some indications that $d_l = 2$ in equilibrium (see, however, R. J. Birgeneau in this issue), is also not less unsettled than the theoretical one, the main problem being the difficulty in reaching equilibrium within reasonable measurement times, as found also by the Monte Carlo simulations. Most magnetic RF experiments have been on the disordered antiferromagnet in a uniform field[3,26,40,49] (see Section 2). The first experiments[50] already showed a marked RF effect, and the Ising transition in three dimensions (3D) was either broadened or consistent with a large negative value of the exponent α. The first neutron scattering results[16,18] had *clearly* shown destruction of long-range order in the RFIM in both $d = 2$ and $d = 3$, which was also seen in further experiments. However, it was not completely clear that equilibrium had been reached with the field. Experiments on Ising-like systems with appreciable x–y interactions, that *may* be thought as having faster equilibration times,[21,22] appear to have some remnant long-range order in 3D with the RF. Birefringence experiments,[17,19] which probe the short-range part of the correlations (much like the specific heat measurements), do not show a broadening of the Ising transition with the RF. This may, however, demonstrate another type of ordering[26,27] and larger field values may be needed. Recent neutron scattering results by Wong and Cable[20] have probed the nonequilibrium behavior and also suggested, *though with a lower resolution*, that long-range order is retained in 3D.

Clearly, it is impossible at this stage to give a completely definitive evaluation of the LCD problem. In this paper we shall review several of the many realizations of the RF model in Section 2 and discuss the various physical arguments, which are rather instructive, in Section 3. Concluding remarks are made in Section 4, with emphasis on the importance of the time-dependent properties. In fact, it is entirely possible that the true equilibrium behavior (as obtained theoretically by calculating the total partition function) may be different from what happens in very long-lived "glassy" metastable states.

2. SOME EXAMPLES FOR RANDOM-FIELD SYSTEMS

1.2. Pinning of the Vortex Lattice; Adsorbed Layers

An obvious realization of an RF coupled to the order parameter is the tendency of defects or impurities to distort or randomly "pin" the periodic arrangement of a crystal. This may happen when a 2D crystal is coupled to the 3D world. The first example of a random-field problem has indeed been the work of Larkin[1] on the destruction of the vortex lattice in type-II superconductors by random pinning. An adsorbed monolayer also experiences a potential energy due to the substrate. In the ideal case, this may be thought of as an "external" periodic (commensurate or incommensurate) potential. However, when the substrate is not perfect (due to defects, corrugations, crystallites, etc.), this potential which couples linearly to the adsorbate order parameter becomes random.[32] This effect should be important in many adsorbate phase transition problems.

2.2. Effect of Regular Impurities on First-Order Transitions

Consider the first-order transition as a function of a uniform magnetic field, H, at $H = 0$ for a ferromagnet below T_c. If an RF is now switched on and if the system is, say, below the LCD, there will be no ferromagnetic phase and the first-order transition as a function of H will be smeared. Above the LCD, the first-order transition will be between the "up" and "down" ferromagnetic random-field phases. This problem is isomorphous to a first-order transition driven by any other intensive variable (e.g., temperature, T) in the presence of impurities which locally influence the first-order transition temperature (or the transition value of the appropriate variable). One thus reaches the conclusion that the effect of regular "T_c" impurities on first-order transitions is analogous to that of a random field on the ferromagnet. In fact, one may consider the energetics of domains of the wrong phase, which is in a one-to-one correspondence with the similar consideration in the random field case. This has been treated in some detail by Imry and Wortis.[35] Since there exist many solid-state first-order transitions, we think that this should be an ubiquitous realization of the random field model. The need to be in the solid state arises in order to have the random variable *quenched*.

2.3. Impurity Effects on Displacive and CDW Transitions

Another example of random fields is provided by impurities whose symmetry allows them to couple linearly[31] to a displacive, ferroelectric,[28-30] or CDW-type[33,34] order parameter. Such off-center impurities have been considered by Halperin and Varma[31] as a model for the central peak often observed in scattering from such systems. If the motion of these impurities can be considered as quenched on the time scales of interest, the central peak associated with them will be almost "elastic." In this case they will generate quenched random fields of the type discussed here.

2.4. Disordered Magnets

Recently, popular examples of random-field systems have been various impure magnetic systems. Imagine a ferromagnetic system, with impurities that are antiferromagnetically coupled to the host atoms. If the impurity spins are classical, it has been suggested by IM that one may redefine them by a factor of -1 to make their interactions with the host ferromagnetic (this is a simple gauge transformation). The application of a uniform magnetic field on this system will result in a negative field on the (redefined) impurities, so that the applied field will have a random component. In the special case where half the host atoms are replaced by impurities—and assuming for simplicity the interimpurity interaction to be ferromagnetic—a pure random field is obtained.

In a very similar fashion, one obtains in simple spin-glass models that can be mapped to ferromagnetic ones by gauge transformations that a uniform physical field is transformed to a random one. An example for this is the Mattis model.[51] This should also apply to more realistic spin glass models, at least when the ordered state is well defined. The uniform field may have an effect similar to an RF on the spin-glass order parameter also in real spin glasses.

Finally, we note that a classical antiferromagnet in a uniform field may be "gauge" transformed to a ferromagnet in a staggered field. If, for example, the site dilution is introduced,[49] the staggered field will be disordered and will act in many respects as a random field (for example, the excess of + fields over − fields in a volume V will be on the order of $V^{1/2}$). In the case of bond dilution,[26] the effective random field will be proportional to the average antiferromagnetic moment, through the exchange interaction. One should note, however, that in these examples the random field appears together with a staggered one as well as with a random exchange. The former is probably less important and the latter is usually argued to be almost irrelevant. However, the correlation between the dilution and the random field is something that has not been addressed so far. For strong dilution, one may also be worried about geometrical and cluster-related effects, especially near the percolation threshold.[52] The exact correspondence of the dilute antiferromagnet in a field to the RFIM should be *closely checked.*

3. PHYSICAL ARGUMENTS

3.1. Bulk Considerations

To establish the instability of the ordered state to an RF, one may consider various mechanisms. It should be kept in mind, however, that demonstrating a particular instability provides only a *sufficient* condition for the inexistence of the ordered state. There may always exist *stronger* instabilities that could be operative.

IM considered the energetics of creating a "wrong" domain in the ordered phase. This is determined by the balance between the possible energy gain due to the RF[53] and the price in energy of the created domain wall. This argument showed that the ordered state is unstable to an arbitrarily small RF below $d = 2$ for Ising (and probably more general discrete symmetry systems) and below $d < 4$ for continuous symmetry (i.e., Heisenberg and x–y) systems. Similar results are obtained using a different language by considering how the RF amplitude h, as well as the exchange interaction, J, behave with length scale L, in a scaling RG approach.[54]

$$J(L) \sim JL^{d-1} \tag{3.1}$$

$$h(L) \sim hL^{d/2} \tag{3.2}$$

where the first equation is the usual scaling of J for the Ising model (related, in fact, to the surface tension[36]). $d - 1$ is replaced, of course, by

$d-2$ in continuous symmetry systems. The second equation[32] expresses the random addition of fields in a volume L^d (as in the IM arguments). Aharony and Pytte[54] obtained it by noting that below the LCD one has something like a first-order transition at $h=0$. (3.1) and (3.2) yield for the relevant dimensionless parameter, which is h/J for (1.1):

$$\frac{h}{J}(L) = \frac{h}{J} L^{(2-d)/2} \tag{3.3}$$

for the Ising-like case. $2-d$ is replaced by $4-d$ for continuous symmetry systems. Thus h/J will increase with L for $d<2$ ($d<4$ in continuous symmetry systems) and will break up the ordering for $L \gtrsim L_c$, where

$$L_c \cong \begin{matrix} (J/h)^{2/(2-d)} & \text{Ising-like systems} \\ (J/h)^{2/(4-d)} & \text{continuous symmetry systems} \end{matrix} \tag{3.4}$$

in agreement with IM. More detailed considerations yield an exponential dependence of L_c on $(J/h)^2$ at the LCD. L_c is the size of the domains to which the system will split[3] due to the RF. These results appear to be correct for the continuous symmetry case, but have been challenged for the Ising case.[4–8] One would like to have a real calculation leading to (3.1), (3.2). Aharony and Pytte[54] also discussed the scaling relations ensuing from (3.3) for the thermodynamic functions and the structure factor.

An important concept for the RF problem is the reduction in the dimensionality. In order to understand this qualitatively,[10] recall the Pippard argument[55] leading to the usual "hyperscaling" law $\nu d = 2-\alpha$ in critical phenomena. The correlation volume ξ^d is the typical unit that can turn into the wrong phase by thermal fluctuations. Since the free energy density price for that goes like $t^{2-\alpha}$ $[t \equiv (T-T_c)/T_c]$, one finds

$$Ct^{2-\alpha-\nu d} \sim kT, \qquad 2-\alpha = \nu d \tag{3.5}$$

where C is a noncritical constant. In the RF problem the principal disordering agent is *not* the temperature but the random field fluctuations. The typical value of the random-field fluctuation in a volume ξ^d is $\sim \xi^{d/2} \sim t^{-\nu d/2}$; coupled with a magnetization t^β, this replaces (3.5) by

$$Ct^{2-\alpha-\nu d} \sim t^{-\nu d/2+\beta} \quad \text{or} \quad (2-\alpha) = \nu(d-2+\eta) \tag{3.6}$$

where we used the usual d-independent scaling law

$$2\beta = (2-\alpha) - (2-\eta)\nu \tag{3.7}$$

Thus, d is replaced by $d-2+\eta$ in the d-dependent scaling laws! This picture also suggests that the critical behavior of the RF model in d dimensions is the same as that of the pure one at $d-2+\eta$ dimensions. We have made this argument only for the (more interesting) Ising-like case. In the continuous symmetry system, Bloch walls will make the spin turn continuously across the domain and may complicate this simple argument.

The value of η which appears in (3.6) is that appropriate to the actual dimension $d' \equiv d - 2 + \eta(d')$. If we ask what dimension corresponds to the LCD of the pure Ising model, $d_l = 1$, at which $\eta(d') = 1$, we find $d = d' + 2 - 1 = 2$. This agrees with the previous arguments[3,32,38,39] that yielded $d_l = 2$ for the RFIM. Equation (3.6) also agrees with the equivalence of the $d = 6 - \epsilon$ RFIM with the $d = 4 - \epsilon$ pure Ising model, to $O(\epsilon)$ *only*. At order ϵ^2, η comes in and makes (3.6) *disagree* with the dimensionality reduction by 2, which has been proven to all orders in ϵ by perturbation theory. This point will be discussed later.

In the theoretical calculations, the propagators of the RFIM are the squares of those of the pure Ising model; this suggests that the usual critical-scattering Lorentzian correlation functions around the Bragg peaks of the ordered phase should be replaced by their squares, which indeed appears to happen in the experiments.[16,18,20–24] The physical mechanism for this, similar to the transverse susceptibility effects calculated for the continuous symmetry case by IM, is simply that the response, M_k, to the RF Fourier component h_k is $\chi_k h_k$, χ_k being the appropriate k-dependent susceptibility, $\chi_k = \chi(1 + k^2\xi^2)^{-1}$; thus, neglecting critical effects,

$$\langle |M_k|^2 \rangle = \frac{\chi^2 h^2}{(1 + k^2\xi^2)^2} \tag{3.8}$$

for k much smaller than the inverse lattice constant. This is the characteristic "Lorentzian squared" line shape. Note also that the crossover to the RF-dominated behavior[3] is fast and is determined by the pure susceptibility exponent γ.[58]

Finally we call attention to the recent numerical studies of the 2D RFIM by Fernandez and Pytte.[48] The exact transfer matrix product was evaluated for finite size RF systems. From this one may obtain the free energy and the susceptibility as well as the correlation function. At very low temperatures (consistent with the expected crossover analysis)[54] one obtains a correlation length that goes like $\exp[(J/h)^2]$. This is consistent with the theories according to which $d_l = 2$. The scaling behavior as function of h and T is also checked and found to be in agreement with Ref. 54.

3.2. Interface Considerations

Since the main physical difference between the effects of RF on the Ising and the continuous symmetry models lies in the sharp domain

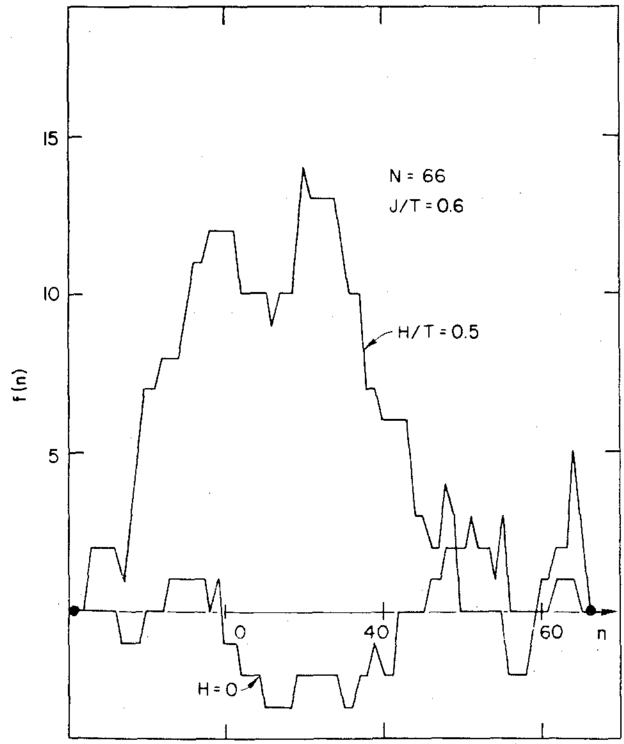

Fig. 1. Snapshots of the SOS model for the Ising interface as seen by a Monte Carlo Simulation with and without an RF (see text for details.)

interface vs. the smeared Bloch wall, it might be thought that interesting effects may follow from the roughening[41,4,5] of the Ising interface. The rough interface thus might yield results intermediate between those of the sharp interface and the completely smeared Bloch wall. This, in fact, becomes more interesting since the RF yields an additional roughening of the Ising interface. While the details of this RF-induced roughening are still at this point under some debate, it clearly exists and has relevant consequences. To illustrate the effect, we show in Fig. 1 the results of a Monte Carlo simulation of a solid-on-solid (SOS) model for the Ising interface. In this model for the 1D interface in a 2D system, fluctuations of the flat interface are described by the column heights f_i, where $i = 0, \ldots, N$ and f_i are integers; with the Hamiltonian

$$\mathscr{H} = 2J \sum_{i=0}^{N} |f_i - f_{i+1}| + 2h \sum_i \sum_{j=0}^{f_i} \epsilon_{ij} \operatorname{sgn}(f_i) \tag{3.9}$$

where the second term represents the random field $h\epsilon_{ij}$ on the (ij)th site of the underlying Ising model. ϵ_{ij} are independent random variables, each distributed uniformly between $-1/2$ and $1/2$. The boundary conditions $f_0 = f_N = 0$ are taken. Figure 1 clearly demonstrates for $J/T = 0.6$, $N = 66$, the very large interface fluctuations induced by $h/T = 0.5$. Figure 1 is taken from unpublished work by Imry and Kirkpatrick.[15]

A simple qualitative argument which is equivalent to the approximate RG calculation of Grinstein and Ma[39] and Villain[32] is as follows. Regarding, for simplicity, f_i as a continuous function $f(x)$ we develop it in Fourier series

$$f(x) = \sum_k f_k \sin kx \tag{3.10}$$

where the allowed k's are determined by the boundary conditions, i.e., $kL/\pi =$ integer, $L = N$. Let us look at the fluctuation f_k for one of the smallest possible k's. The details of what happens depend somewhat on the precise assumptions on the model. For the continuum case, assuming the interface fluctuation to cost an energy $\sigma\int_0^L(\nabla f)^2 = (\sigma L k^2/2)f_k^2$, for the given k, where the surface tension, σ, is proportional to J, the number of spins involved in this fluctuation is on the order of $f_k L$ so that the possible energy gain is on the order of $h(f_k L)^{1/2}$. Minimizing the sum of these two energies with respect to f_k yields

$$\langle f_k^2 \rangle \sim (h/\sigma)^{4/3} L^{2/3} k^{-8/3} \tag{3.11}$$

The total fluctuation $\langle f^2 \rangle$ (say, at the middle, $x = N/2$) is dominated for low d by $k = \pi/N$ (so that one should not worry too much about the possible lack of independence of the various k's, etc.), so that

$$\langle f^2 \rangle \sim (h/\sigma)^{4/3} L^2 \tag{3.12}$$

For any finite h this field-induced roughening is much larger, for $L \to \infty$, than the usual thermal roughening which yields $\langle f^2 \rangle \propto L$, for $h = 0$ at $d = 2$. The coefficient of L^2 depends on the details of the model (whether continuum or discrete, $|\nabla f|$ or $|\nabla f|^2$ interactions), but the power of L, which turns out in general d to be $2(5-d)/3$, stays unchanged. In Fig. 1 one can notice the importance of the few small k components.

The LCD is obtained in this picture as the d below which $\langle f^2 \rangle \gg L^2$ as $L \to \infty$. This clearly means that finite domains with $f \sim L$ will spontaneously form below the LCD, which is $d_l = 2$ in this model. Taking the model yielding $\langle f^2 \rangle \sim (h/J)^{4/3} L^{2(5-d)/3}$, we find for the domain size

$$L_c \sim (J/h)^{2/(2-d)} \tag{3.13}$$

which happens to be the same as the bulk result (3.4). Different interface models may yield different factors multiplying $1/(2-d)$ in the exponent.[40,43] For $d=2$ more careful treatments yield L_c which is exponential in $(J/h)^2$.

The additional strong roughening of the domain wall in the presence of the RF is an effect interesting in its own right. According to the discussion in Section 2 this should occur in *any* situation of a two-phase equilibrium in the solid state, in the presence of impurities. This should be observable by x-rays, neutron, or light scattering from this interface. Consider, for example, light scattering from such an interface with a difference Δn in the refractive indices of the two phases, and with a momentum transfer, **K**, parallel to the interface. A straightforward calculation yields for the scattering intensity:

$$S(K) \propto (\Delta n)^2 \langle f_K^2 \rangle \tag{3.14}$$

which can be used to determine the dependence of $\langle f_K^2 \rangle$ on K, h, and L. This is of interest both above and below the LCD.

The additional RF interface roughening (below $d=5$, in theory) was first discussed by Pytte, Imry, and Mukamel,[4] who found, from a replica-trick calculation

$$\langle f_k^2 \rangle \sim (h/J)^2 k^{-4} \tag{3.15}$$

This yields at $d=2$ an exponent of 3 in (3.12), and $5-d$ in general $d<5$, in disagreement with the qualitative consideration given above. This necessitates using $L^{(d-1)/2}f$ instead of $L^{(d-1)/2}f^{1/2}$ in the expression for the energy gain due to the field,[4,5,6] which may not be correct.[56] It would follow were one using force instead of energy consideration, which may apply as long as f is less than the interface width. More definitive work on this is needed.

Computer calculations[43] on the interface at $d=2$, using a transfer matrix method, support (3.12) (or variations thereof[40] due to the details of the model which preserve the powers of L) and not (3.15). Previous Monte Carlo calculations[15] on the same problem could not distinguish sharply enough between the two possibilities. The latter calculations pointed out, however, that extremely interesting dynamics exist for this problem, perhaps in analogy to the spin-glass case. In fact, the relaxation or equilibrium times of the small k interface modes increases sharply with k^{-1} or with L.[15,57] This is easy to understand in terms of free-energy barriers among metastable states. This possible existence of metastable states in the RFIM is an important aspect that has also to be taken into account in the experiments.

4. CONCLUSIONS AND REMARKS

We hope to have shown in this paper some of the many physical realizations of the RF model as well as the interesting questions it poses. At this stage we have formal theories relying either on perturbation expansions or on analyticity and replica symmetry, in strong disagreement with appealing physical arguments, approximate calculations, and finite size numerical results. Hopefully, this question (as well as the possible effects of dilution) will be resolved soon, both experimentally and theoretically. However, the question of what is the LCD, and what is the behavior of the RFIM at the physical dimension $d = 3$, is *not* the only interesting one. One should really focus also on the very interesting slow relaxation and metastability effects that show up in these systems, and which greatly complicate the experimental studies. Different behavior in equilibrium and in metastable states is possible. We also emphasize the possibility of directly studying the RF interface fluctuations (perhaps including their dynamics) by scattering experiments. Finally, we mention the very interesting case of an RF interface where the RF is annealed in one of the phases and it is quenched in the other. An example of this is the impure crystal–fluid interface, which shows extremely interesting separation and segregation effects.

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